tūturu

Wellbeing Teaching and Learning Guide

NCEA Level 2

Mathematics and Statistics

2.9 Statistical Inference

Internal Assessment Resource

Achievement Standard 91264

Use statistical methods to make an inference

4 credits

Contents

Achievement Standard 91264

Subject Reference		Mathematics and Statistics 2.9			
Title		Use statistical methods to make an inference			
Level	2	**Credits**	4	**Assessment**	Internal
Subfield		Statistics and Probability			
Domain		Statistics			
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This achievement standard involves using statistical methods to make an inference.

Achievement Criteria

Achievement	Achievement with Merit	Achievement with Excellence
Use statistical methods to make an inference.	Use statistical methods to make an inference, with justification	Use statistical methods to make an inference, with statistical insight.

Explanatory Notes

1. This achievement standard is derived from Level 7 of *The New Zealand Curriculum* (2007), and is related to the material in the *Teaching and Learning Guide for Mathematics and Statistics*, Ministry of Education, 2010 at http://seniorsecondary.tki.org.nz. The achievement standard is aligned to the following achievement objectives taken from the Statistical Investigation thread of the Mathematics and Statistics learning area:
    - carry out investigations of phenomena, using the statistical enquiry cycle:
        - using existing data sets
        - evaluating the choice of sampling and data collection methods used
        - using relevant contextual knowledge, exploratory data analysis, and statistical inference
    - make inferences from surveys:
        - using sample statistics to make point estimates of population parameters
        - recognising the effect of sample size on the variability of an estimate

This standard is also derived from *Te Marautanga o Aotearoa*. For details of the *Marautanga* achievement objectives to which this standard relates, see the Māori version of the standard.

2. *Use statistical methods to make an inference* involves showing evidence of using each component of the statistical enquiry cycle to make an inference.

   Use statistical methods to make an inference, with justification involves linking components of the statistical enquiry cycle to the context, and/or to the populations, and referring to evidence such as sample statistics, data values, trends, or features of visual displays in support of statements made.

   Use statistical methods to make an inference, with statistical insight involves integrating statistical and contextual knowledge throughout the statistical enquiry cycle which may involve reflecting on the process, or considering other explanations.

3. Using the statistical enquiry cycle to make an inference involves:
   - posing an appropriate investigative comparison question from a given set of population data
   - selecting random samples
   - selecting and using appropriate displays and measures
   - discussing sample distributions
   - discussing sampling variability, including the variability of estimates
   - making an inference
   - communicating findings in a conclusion.
4. Conditions of Assessment related to this achievement standard can be found at http://ncea.tki.org.nz/Resources-for-Internally-Assessed-Achievement-Standards.

New Zealand Curriculum Achievement Objectives

Achievement Objective S7-1

In a range of meaningful contexts, students will be engaged in thinking mathematically and statistically. They will solve problems and model situations that require them to:

- Carry out investigations of phenomena, using the statistical enquiry cycle:
  - conducting surveys that require random sampling techniques, conducting experiments, and using existing data sets
  - evaluating the choice of measures for variables and the sampling and data collection methods used
  - using relevant contextual knowledge, exploratory data analysis, and statistical inference.

Achievement Objective S7-2

In a range of meaningful contexts, students will be engaged in thinking mathematically and statistically. They will solve problems and model situations that require them to:

- Make inferences from surveys and experiments:
  - making informal predictions, interpolations, and extrapolations
  - using sample statistics to make point estimates of population parameters
  - recognising the effect of sample size on the variability of an estimate.

Introduction

This unit of work gives students the opportunity to work at Level 7 of the NZ curriculum, focussing mainly on Achievement Objective S7-1 and S7-2, which is primarily aimed at using statistics to make an informal inference on a population.

The unit focusses on our generic heading of 'Wellbeing', which is a term used to describe the level of wellness or condition of an individual. While the oxford dictionary defines wellbeing as 'the state of being comfortable, healthy, or happy', there are many variables that contribute to a positive or negative level of wellbeing;

- home environment and support from home
- school environment and support whilst at school
- community links and communal activities such as church, sports etc
- strong links to friends and family
- substance abuse, if any

Measuring wellbeing is challenging as there are a number of different variables to take into account. It can be done in a number of ways and there is no 'one size fits all' approach. The largest study of its kind in New Zealand, has been recently completed in 2012 by the Adolescent Health Research Group, in an attempt to look into the wellbeing of young people in New Zealand aged between 13 and 17. The data has been made available for discussion and analysis and you will be exploring this data in this unit of work.

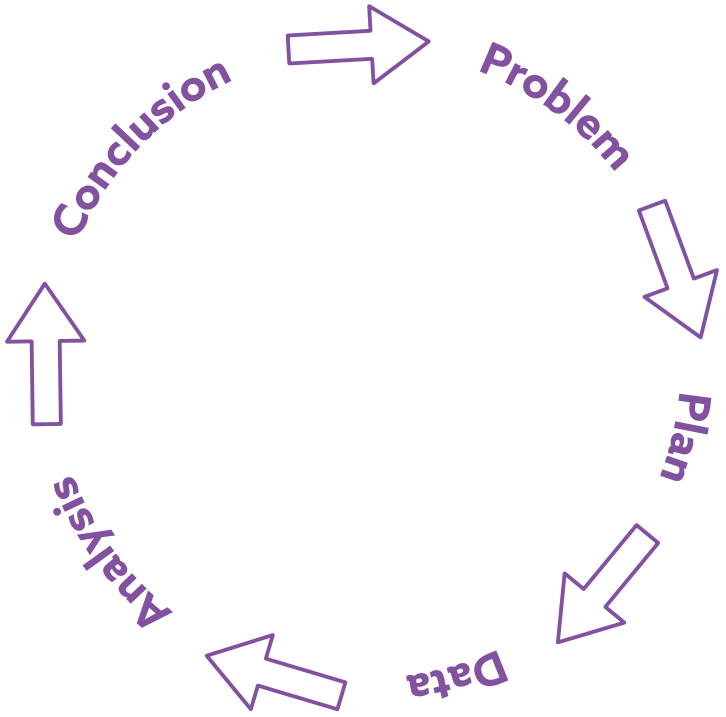
This unit focuses on a few of the variables from this study in an attempt to see a snapshot of the lives of young people aged 13 to 17 in New Zealand.

Questions

1. "This unit focuses on a few of these variables in an attempt to see a snapshot of the lines of young people ages 13 to 17 in New Zealand."
   a. **What do you think the word 'snapshot' means in this sentence?**
   b. **What other variables can you come up with, that would contribute to a positive or negative 'wellbeing' level?**

The Statistical Enquiry Cycle

The statistical enquiry cycle is a cycle that is used to carry out a statistical investigation. The cycle consists of five stages: Problem, Plan, Data, Analysis, Conclusion. The cycle is sometimes abbreviated to the PPDAC cycle.



1. Problem

The problem section is about formulating a statistical question, what data to collect, who to collect it from and why it is important. What is it that you want to measure?

2. Plan

The plan section is about how the data will be gathered. Look at how you can solve the problem. How are you going to measure it? Collect the data.

3. Data

The data section is about how the data is managed and organised. Clean and sort the data.

4. Analysis

The analysis section is about exploring and analysing the data, using a variety of data displays and numerical summaries, and reasoning with the data. Make tables and graphs. Look for patterns. Explore what is going on.

5. Conclusion

The conclusion section is about answering the question in the problem section and giving reasons based on the analysis section. Answer the question. Explain what this means. This may lead to other questions.

1. Problem

In this standard you are making an inference about a population. The process involves taking a sample from the population, analysing the sample and using the analysis to make a comparison between the means or medians of two different groups in the population. The problem is a comparison question (comparing one group with the other) and needs to be worded to include:

- a clearly identified population
- what groups are being compared
- what variable is being investigated
- what parameter will be used to make the comparison
- the direction of the comparison, i.e. greater than or less than

Example

I wonder if the **median** time teenagers spend on mobile phones is greater than the **median** time adults spend on mobile phone from an online study in Australia in 2015.

The **population** identified in this investigation is those who participated in the online study in Australia in 2015

The **groups** being compared are teenagers and adults

The **variable** being investigated is the time spent on the mobile phone

The **parameter** used to make the comparison is the median. It is important that the parameter is included in the question as it can sometimes be left out unintentionally. It is important because the comparison being made is between either the mean or median of the two groups. Generally the median is the preferred choice for this standard.

The **direction** of the comparison is whether the teenager median is greater than the adult median.

Developing Your Problem

The quality of your report depends on how well you tie in relevant contextual knowledge into the writing of the report. Much of this can take place at the beginning by way of introduction. This helps inform the purpose of the investigation and give relevance to the question you have posed. If the purpose of the investigation is not clear it may be helpful to look into the background information

surrounding the data set and context to find out more.

Consider why you are interested in variables you have chosen to investigate. Background research into the variables will help you identify clearly what they mean, what they are measuring and how they are measured.

A good problem section of your report has the following:

- An introduction to the data set, its context and the variables of interest
- Some research or discussion of the variables relevant to the context with possible links to further research
- The reason for investigating the variables – this guides the purpose of the investigation
- The comparison question worded correctly
- A hypothesis or prediction of what you think the answer is likely to be

Questions

1. I wonder if the median wellness score for female students is greater than the median wellness score for male students who took part in the New Zealand Youth Health and Well-being survey in 2012.
   a. **What is the population identified in this investigation?**
   b. **What are the groups being compared?**
   c. **What is the variable being investigated?**
   d. **What is the parameter used to make the comparison?**
   e. **What is the direction of the comparison?**

Answers:

a. *those who participated the New Zealand Youth Health and Well-being survey in 2012*

b. *females and males*

c. *the wellness score*

d. *the median*

e. *whether the female median is greater than the male median.*

2. Plan

In the plan you need to include the following in your report:

- the variables being investigated including units
- sampling method and sample size
- justify the sampling method used
- discuss where the data has come from

Sample vs Population

A population consists of all elements from a set of data, whereas a sample contains only a part of a population. While the most accurate conclusions can be drawn from having access to all the data from the population, this is often not possible. Therefore a sample is taken and from that sample, we can make a **statistical inference** about the population.

There are different kinds of sampling methods that can be used. Some are more appropriate to use at times than others depending on the nature of the data set. It is generally accepted that a minimum sample size of 30 is required to make a statistical analysis, although the more data we have, the greater the precision of our results and the lower the variation. We want to take a big enough sample size, so that the data is accurate enough to represent the population.

In this standard, you will need to take a sample from a larger dataset just as you would from the population and justify the method chosen.

Random Sampling Methods

We want samples to be selected randomly so the characteristics of the sample are typical (representative) of the population. A random sample means that each member of the population has the same chance of being selected. A biased sample is not typical of the population and has a bias for particular members.

1. **Simple Random Sampling**

   This is a simple and straightforward method in which each individual is selected entirely at random.

   + *each member of the population has an equal chance, or probability, of being selected.*
   - *a group may be under-represented in the population, meaning it will also be under- represented in the sample. e.g., for hospitalised diabetes patients, most suffer from hyperglycaemia (high blood sugar) with hypoglycaemia*

(low blood sugar) being uncommon. In the sample most would be hyperglycaemic with only a few being hypoglycaemic.

2. **Systematic Sampling**

   A systematic sample involves selecting individuals at regular intervals from the population and is particularly useful if you have access to a list of all individuals. For instance, every 10th student might be chosen from the school roll.

   + *easy to administer.*
   - *may lead to bias if there are underlying patterns in the order of the individuals. e.g., if people are listed by alternating male and female then every 10th individual selected (because even) would only be female.*

3. **Stratified Sampling**

   In the stratified sampling method, the population is first divided into subgroups who all share a similar characteristic. Then a specified number of individuals (usually the same for all) is randomly chosen from within each subgroup.

   + *good for when one group is under-represented in the population. e.g., ethnic minorities.*
   - *it can be difficult to decide which characteristic(s) to stratify by.*

4. **Clustered Sampling**

   In a clustered sample, subgroups from the population are used as the sampling units. These are known as clusters and are generally already defined in the population e.g., schools, shopping centres, towns. In single-stage cluster sampling, all members of the chosen clusters are included. In two-stage cluster sampling, a selection of individuals from each cluster is randomly selected.

   + *More efficient. e.g., it is easier to sample students from one school than it is to sample 600 students from all across New Zealand.*
   - *Increased risk of bias if the cluster is not representative of the population. e.g., students sampled from two boys high schools in Wellington.*

The New Zealand Youth Health and Well-being Survey 2012

In 2012, a two-stage random sample was completed to collect a survey population of 2,996 students.

Stage One

Firstly 125 schools were randomly selected throughout the country to partake in the survey.

Stage Two

Of schools that opted into the survey, 20% of the students were randomly selected to complete the survey. If the school had less than 150 students, 30 students were

selected to ensure a viable group has been selected and also to ensure privacy was at its maximum.

These students were invited to participate in the New Zealand Youth Health and Well-being Survey 2012, one in which parental permission had to be obtained, and of which only 2,996 students responded. The population for this investigation is the Participants of the New Zealand Youth Health and Well-being Survey 2012. Students completed a questionnaire on a hand-held tablet, allowing questions to be presented in an audio-visual form.

In this standard, you will need to obtain a suitable sample from this population in order to make an inference.

Questions

1. "In 2012, a two-stage random cluster sample was completed to collect a population of 2,996 students. Stage One: Firstly 125 schools were randomly selected throughout the country..."
    a. **Would this sampling technique exclude any young person aged between 13 and 17?**
    b. **How would the researchers have come up with a sampling frame (a list of all schools in New Zealand) and how would they have used this list to select the 125 schools randomly?**

2. "Stage Two: Of the schools that opted into the survey, 20% of the students were randomly selected to complete the survey."
    a. **How would the school / researchers obtain a random sample of 20% of the schools roll?**
    b. **Why was cluster sampling a good technique to use?**
    c. **Could any of the other sampling techniques (simple random sampling, systematic, stratified sampling) worked to gather the population?**

3. "If the school had less than 150 students, 30 students were selected to ensure a viable group has been selected and also to ensure privacy was at its maximum."
    a. **Why do you think the researchers did this?**

4. Every sampling technique has advantages and disadvantages.
    a. **Discuss ways this technique may have gathered a population dataset that is or is not representative of the population of all young people aged between 13 and 17 in New Zealand?**

3. Data

Data may need to be cleaned to remove any inappropriate or missing values. Before using a data set:

- check that it contains the data we expect
- clean the dataset by removing blanks or unexpected values

The New Zealand Youth Health and Well-being Survey 2012 Dataset

Variable	Description
Gender	Male or Female
Age (years)	The age of the student rounded down to the closest whole year
Wellbeing Score	This variable is the World Health Organisation-5 Wellbeing Index (WHO-5). Scores can range from between 0 to 25. The larger the WHO-5 score, the more likely they felt cheerful, calm, active, rested, and had things in their life that interested them over the previous two weeks.
Feelings	This variable is called the RADSSF score. Scores can range from between 10 and 40. The larger the RADSSF score, the more likely the student experienced low moods.
Challenges	This variable is the Total Difficulties Score from the Strengths and Difficulties Questionnaire. Scores can range from between 0 to 40. The larger the score, the more likely they had experienced social and emotional challenges over the past six months.
School Connection	This variable is the school connection score. Scores can range between 0 to 4. The larger the score, the more likely they had felt connected to their school.
Family Connection	This variable is the family connection score. Scores can range between 0 to 4. The larger the score, the more likely they had felt connected to their family.

Family Meals	This variable is the response to the question: "*During the past 7 days, how many times did all, or most, of your family living in your house eat a meal together?*" Responses were given the following values: Never = 1 1-2 times = 2 3-4 times = 3 5-6 times = 4 7 or more times = 5
Binge Drinking	This variable indicates that the student reported binge drinking once or more in the last month. Binge drinking was defined as 5 or more alcoholic drinks within a 4 hour period. Not at all = 0 Once or more = 1

Questions

1. Researchers have spent the last few years analysing the data ready for release to the public.
   a. **The study was conducted in 2012. Would there have been any / many changes since then?**
   b. **Which of these variables do you believe would be changing the greatest for young people living now?**

2. One of the groups that could be used in the investigation is gender;
   a. **How critical is gender in this investigation?**
   b. **How different are teenage males to teenage females, with these variables in mind?**
   c. **Which of the variables would be greatest (or least) affected by gender?**

4. Analysis

Once samples have been taken from the population, the data needs to be analysed.

You will need to;

Use appropriate displays:

- Dot Plot and/or
- Box and Whisker Graph

Calculate appropriate sample statistics:

- Central Tendency (mean, median)
- Spread (range, IQR, standard deviation)

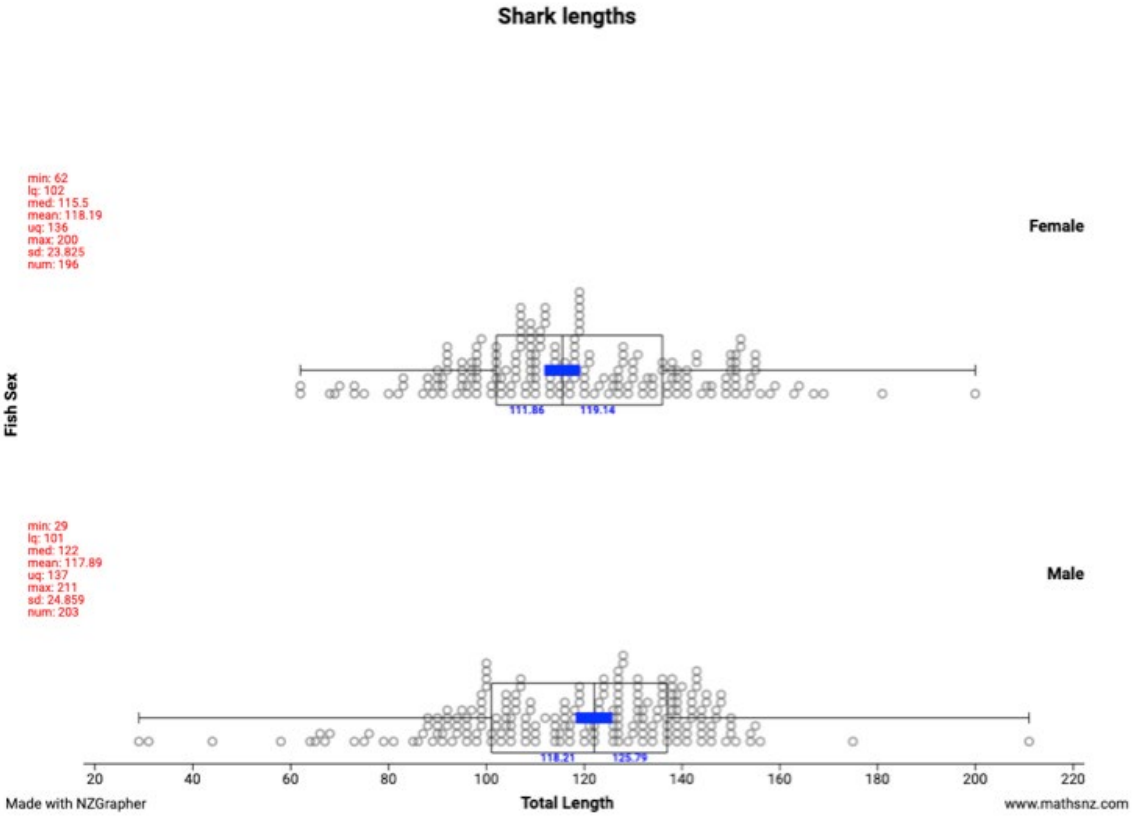
Discuss and compare the sample distributions. It is important that you compare two groups. It is not enough to simply describe the features of the two groups separately:

- Centre
- Shape
- Spread
- Unusual Features

Display

The most useful graphical display for this standard is the dot plot with box and whisker graph. There are a number of suitable software packages that can do this - popular are the online resources iNZight and NZGrapher. The dot plot with box and whisker graph below has been created with NZGrapher. It shows the summary statistics on the left hand side and includes bars across the median lines to represent the informal confidence interval for each group.

Discuss and Compare the Sample Distributions



Feature	Discussion
Centre	Compare the centre of each group by identifying the medians and saying which is greater. Justify by calculating the difference so you can show how much they differ by.
Shape	You will need to compare by describing the overall shape of the data for each group. Use the following keywords to describe: ▪ Normal distribution (symmetrical, bell shaped curve) ▪ Left skewed (tail on the left side) ▪ Right skewed (tail on the right side) ▪ Multi modal (more than one peak) ▪ Uniform (rectangular shaped, generally all one level) Justify the shape chosen by thinking about the following features: ▪ symmetry ▪ the tail that appears either end ▪ how many peaks are visible

Spread	Compare the spread of the data in each group by using any of the following: ▪ the range (distance from the minimum to the maximum data values) ▪ the interquartile range IQR (distance from the lower quartile to the upper quartile) ▪ the standard deviations Justify by calculating the difference so you can show how much they differ by.
Unusual Features	If there is anything unusual in the data from either group, point this out. ▪ outliers (datapoints that lie out at the extremes) ▪ large clusters that stand alone ▪ significant gaps that may be apparent in the data

Questions

1. How well can you link the analysis to relevant contextual knowledge about student wellbeing
   a. **How do the features you have described fit in with your understanding of student wellbeing? Do they make sense?**
   b. **Is there information that supports or contradicts your results so far?**

5. Conclusion

Clearly communicate your findings in a conclusion, linking your findings to the context and population. A conclusion looks back to the Problem and attempts to answer the investigative question and this involves making an inference about the population.

Using information from the informal confidence interval you will determine if there is enough evidence to say that one population median is larger than the other.

Your conclusion needs to include the following:

- Make a supported inference about the population
- Use this inference to answer your problem – 'making a call'
- Discuss sampling variability, including variability of estimates. Link this to the sampling technique you chose, and the inference you have made.

You may also wish to include the following:

- link back to your prediction
- link your conclusion to relevant contextual knowledge
- how could the investigation be extended?
- what assumptions, if any, have been made?
- what could this inference lead on to next?

Informal Confidence Interval

Based on the median of our sample we are able to calculate an interval that the population median is likely to be within. This interval is called an informal confidence interval.

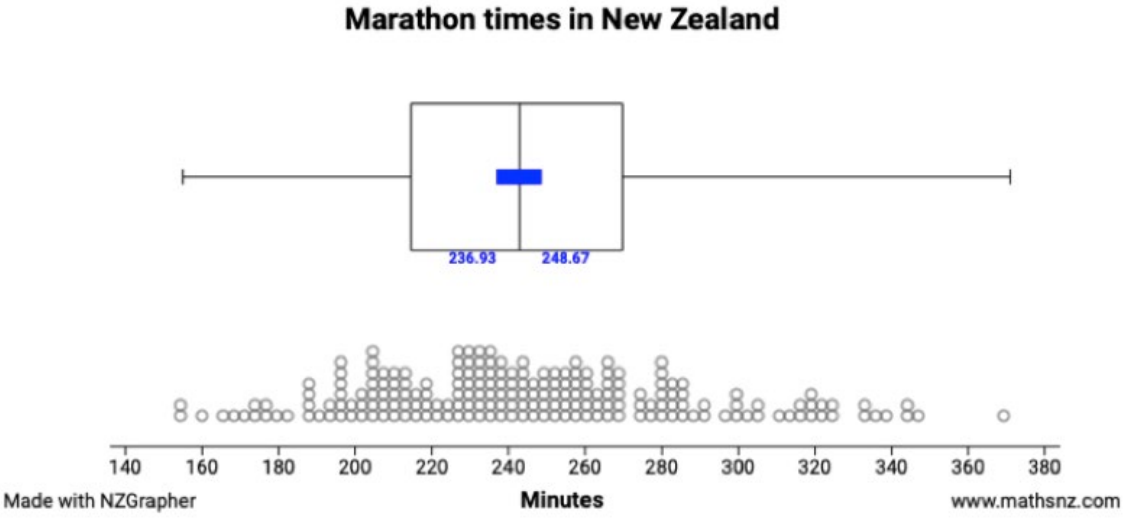
The following formula $1.5\times\frac{IQR}{(\sqrt{n})}$ involves the *IQR* (interquartile range) and *n* (the sample size for the group). This is added to and subtracted from the median to calculate the interval. Therefore the informal confidence interval (ICI) is defined by:

$$\text{median} \pm 1.5\times \frac{IQR}{(\sqrt{n})}$$

You can now make an inference to the population using the ICI. The population median is likely to be somewhere in this interval. The larger the sample taken, the smaller the interval will be and consequently the closer the sample median will be to the actual population median.

Example 1

From the following sample of marathon times in New Zealand displayed below, what inference can be made about the population median?

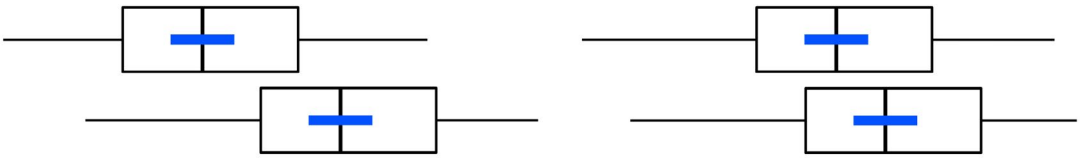


The median marathon time from the population of marathon times in New Zealand is likely to be between 236.9 to 248.7 minutes (or 236min 55sec to 248min 40sec).

Making the Call

Look at the confidence intervals that you have created using the formula or are shown displayed on the graph.

- If they overlap, the medians are too close together and you can't make the call.
- If they don't overlap, the medians are far enough apart so that you can make the call.



Can make the call

Can't make the call

Example 2 NO OVERLAP OF ICI

"In the population of those who participated in the online study in Australia in 2015, the median time spent on mobile phones by teenagers is likely to be between 6.7 to 7.5 hours per day and the median time spent on mobile phones by adults is likely to be between 4.2 to 5.5 hours per day.

For the people who participated in the online study in Australia in 2015, the median time spent on mobile phones per day by teenagers is greater than the median time spent on mobile phones per day by adults."

Example 3 OVERLAP OF ICI

"In the population of those who participated in the online study in Australia in 2015, the median screen time on electronic devices by teenagers is likely to be between 10.7 to 13.5 hours per day and the median screen time on electronic devices by adults is likely to be between 9.8 to 11.5 hours per day.

"For the people who participated in the online study in Australia in 2015, ***it is not possible to make the call*** *that the median screen time on electronic devices by teenagers is greater than the median screen time on electronic devices by adults."*

Note: it is incorrect to say there is 'no' difference, only that the call can't be made.

Sampling Variability

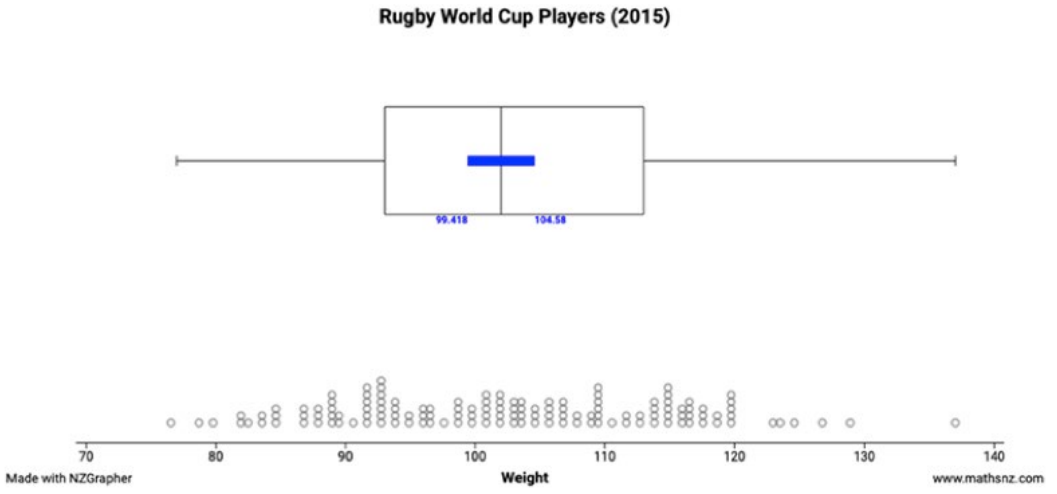
Sampling variability is how much an estimate varies between samples. In general, data values and estimates for population parameters will change from sample to sample especially if that sample is small.

For example, suppose a sample is taken and the mean is calculated. If a second sample of the same size is taken from the same population, it is very likely that the mean calculated from this sample will be different from that calculated from the first sample. If further means are calculated, by repeatedly taking samples of the same size from the same population, then the differences in these means illustrate sampling variation.

Larger samples will contain less sampling variability. For example a sample size of 10 people taken from the same population of 1,000 will give you a very different result than a sample size of 200.

Questions

1. From the following sample of Rugby World Cup players displayed below, what inference can be made about the population median?



**Answer: The median weight from the population of rugby world cup players in 2015 is likely to be between 99.4kg to 104.6kg.*

2. "Students need to obtain a suitable sample from the 2,996 student responses in order to make a suitable inference for the survey population"
   a. **If the sample is a good representation of all 2,996 students that completed the survey, could your inference be a good representation of the survey population?**
   b. **Could it be a good representation for all young people aged 13 to 17 in New Zealand? Why or why not?**

3. "This study was completed in 2012 in order to look into the 'wellbeing' of young people in New Zealand aged between 13 and 17."
   a. **Who or what organisations might benefit from the results of this survey and the findings from your report?**
   b. **Are the findings from this 2012 survey still relevant today? If so why? If not, why not?**

Suggested Teaching Sequence

The teaching sequence for this unit of work has three phases:

<table>
<tr><th>Phase</th><th>Time</th></tr>
<tr><td>Phase 1</td><td>2-3 weeks</td></tr>
<tr><td colspan="2">Students refresh prior learning and develop the Level 7 understandings that will be assessed by Achievement Standard AS91264 (Use statistical methods to make an inference);

They should do this in a variety of contexts other than student wellbeing. The following is a suggested outline:

▪ Statistical Enquiry Cycle review (PPDAC):
▪ Posing a comparison question
▪ Planning: sampling techniques:
- sample size
- advantages and disadvantages
▪ Data: cleaning & collection
▪ Analysis: summary statistics and sample distributions:
- centre, shape, spread, overlap and special or interesting features.
▪ Informal confidence intervals for the population median (overlap)
▪ Conclusion
- inference and ‘making the call’
- sampling variability in relation to the inference</td></tr>
<tr><td>Phase 2</td><td>1 week</td></tr>
<tr><td colspan="2">Set a formative assessment task for learners to work through and use this as a vehicle to check their understanding of the key ideas and to provide specific feedback. The context for this task should be relevant and meaningful to the learners. Support should be provided for students during this practice phase, which provides an opportunity to identify and address any misconceptions or areas of weakness.</td></tr>
<tr><td>Phase 3</td><td>1-2 weeks</td></tr>
<tr><td colspan="2">Students are introduced to the context of student ‘wellbeing’. Students work with the data set to:
▪ clarify what the variables represent
▪ decide on the purpose for their investigative question
▪ select the variables they want to work with
▪ pose an appropriate investigative comparison question.
Students complete and submit the assessment task.</td></tr>
</table>

Help Organisations

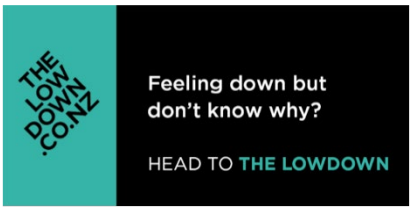
Help for the Tough Times

Help for the Tough Times provides a quick guide to four New Zealand websites that were especially designed to support teens with issues like anxiety, stress, identity, relationships, and depression. A pocket guide was co-developed for teens by teens (by two Year 12 high school classes). Support material is also available for school staff so that they are aware of the websites.

Order FREE hard copies from this website
www.hpa.org.nz/education/help-for-the-tough-times

The Lowdown

www.thelowdown.co.nz
Free phone 0800 11 17 57 or free text 5626



Youthline

www.youthline.co.nz
Free phone 0800 37 66 33 or free text 234

AlcoholDrug Helpline

www.alcoholdrughelpline.org.nz

AuntyDee

www.auntydee.co.nz

Sparx

www.sparx.org.nz

RainbowYOUTH

www.ry.org.nz

Mental health and addictions helpline

Free phone 1737 and free text 1737